**Report outline:**

1. Introduction:
   1. What’s a wave motion and what’s a water wave problem; the relevance and the difficulty;
   2. Introduce the well-known results on the whole line (wave, KdV equations)
   3. Mention the contribution of capstone – derivation of approximate equations on the whole line
2. Preliminaries + Literature review:
   1. Explain the physical assumptions made in the water-wave problem (and justify them in the context of water waves), Lannes 2013
   2. Explain the shallow water regime and why we care about it:
   3. Explain what’s a wave equation, KdV, discuss their history, importance
   4. Explain time scales & explain up to what time the expansion is valid
   5. Explain why we can trust asymptotic models (i.e. justify that solutions of asymptotic model are asymptotic to actual solutions in the shallow water)
3. Derivation from non-local formulation on the whole line
   1. Explain why we care about the non-local formulation (what are the pros, cons, etc) in general:
      1. Simplify equations of motion
   2. Explain why we care about our particular formulation, Oliveras & Vasan 2013
      1. In physical variables, derived without approximation
      2. works for both 1 and 2 dimensional surfaces
      3. in eta only, so that’s good for applications ,
   3. Derive the non-local formulation (use Report 2)
   4. Derive the surface expression (use Report 2)
   5. Derive wave and KdV equations (use Report 2)
      1. Make sure to explain that equations are non-dimensional, and so as we dispense with epsilon, the relationship between equations is hidden
   6. Provide justification for the model: why do we care about obtaining wave and KdV from this formulation?
   7. Discuss alternative derivations and pros and cons (if any). Alternative derivations:
      1. (Ablowitz et al 2006) – the derivation is in velocity potential only
      2. (BBM 1972) – the KdV is derived by combining approximate dispersion and nonlinear equation, but there is no reason to assume that dispersion and nonlinearity have such a balance
      3. DNO (Craig & Sulem 1993) – approximate equations are not derived
4. Future directions:
   1. Discussion of the half-line problem and associated difficulty (emphasise the open-ended nature of the problem)
   2. Moving to bounded domains

**References:**

Oliveras, Vasan 2013, A new equation describing travelling water waves

Ablowitz, Fokas, Musslimani 2016, On a new nonlocal formulation of water waves

Craig, Sulem 1993, Numerical simulation of gravity waves

David Lannes 2013 The water waves problem

Ablowitz 2011, Nonlinear dispersive waves